4 Executive Summary

MV-algebras were proposed by Chang in 1959 [7] as the algebraic counterpart of infinite-valued Łukasiewicz logic, providing a new proof for the completeness of the Łukasiewicz axioms. Since then, the study of these algebras has yielded various results, revealing connections to other algebraic contexts as well as important applications.

Recently, the idea of equipping MV-algebras with a topology has been explored. In 2016, Russo [23] introduced the notion of a fuzzy topology for MV-algebras and described several theoretical properties analogous to classical topology. Building on this work, De La Pava provided further topological notions and results from both classical and fuzzy theory. We propose to continue this line of research, extending the known theory of MV-topologies.

In the study of MV-topologies, Russo defined the concept of limit cut complete MV-algebras and provided an extension of the Stone duality to fuzzy topologies and MV-algebras, among other results. We aim to conduct a detailed study of these structures, examining their elements and algebraic properties.

Furthermore, we aim to investigate topological notions and properties that are of natural interest in the context of MV-topologies, such as convergence, normality and uniformity. Although normality and uniformity have already been described in this context, further applications of these concepts could be examined.

The notion of open regular sets has significance in the context of classical topology. In particular, there is a relation of such objects with the de Vries algebras. We propose to study this notion to describe the structure that is obtained and its possible relation with de Vries algebras.

12 References

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